



National Aeronautics and  
Space Administration  
Langley Research Center  
Hampton, VA 23681-0001



Program 3 in the 1999-2000 Series

# Geometry of Exploration:

## Water Below the Surface of Mars?



■ learn about geometry and planetary orbits



■ use the internet to conduct a WebQuest



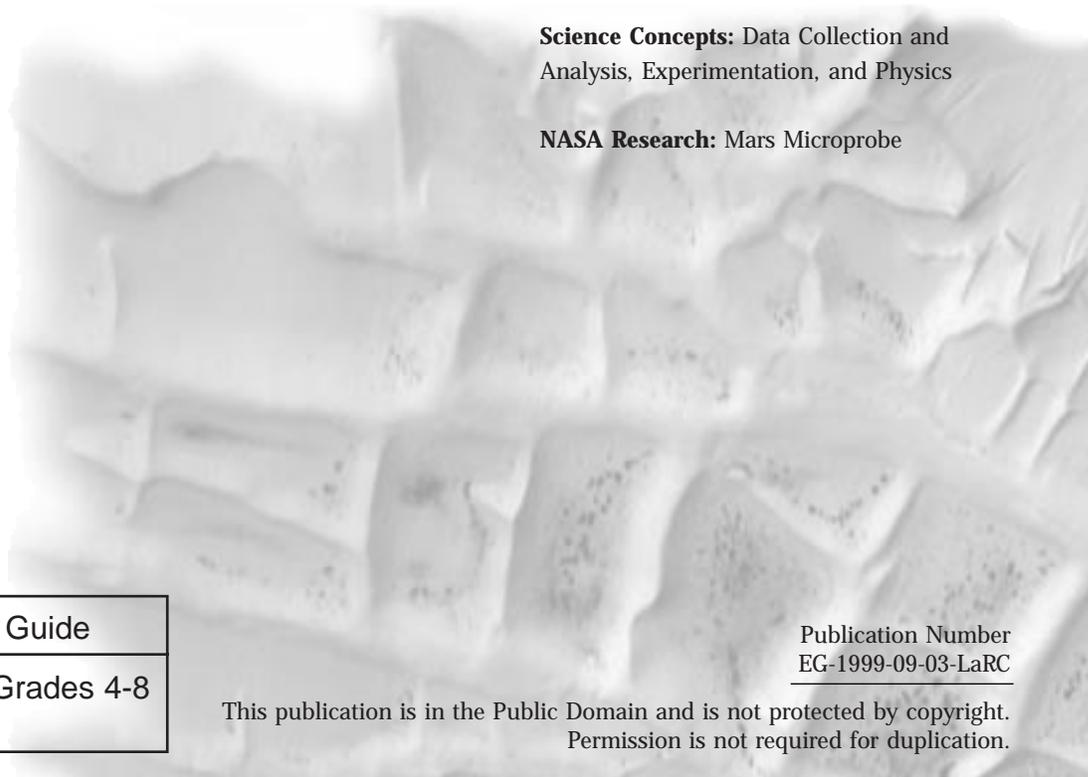
■ use a calculator or spreadsheet

**Story line:** Students will learn how engineers and scientists will use geometry in the solar system to navigate a spacecraft to Mars.

**Math Concepts:** Angles, Graphic Methods, Geometric Shapes, Inference, Reasoning

**Science Concepts:** Data Collection and Analysis, Experimentation, and Physics

**NASA Research:** Mars Microprobe



Educator's Guide	
Teachers & Students	Grades 4-8

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## PROGRAM SUMMARY

### OBJECTIVES

In *Geometry of Exploration: Water Below the Surface of Mars?*, students will learn how geometry, geometric shapes, and navigation are used to explore Mars. They will also join NASA engineers and scientists who seek to answer the age-old question, "Is there water below the Martian surface?" From the Geometry of Exploration program, students will (1) be introduced to geometry, to the theories and contributions of Pythagoras, and will learn how his contributions are being used to explore and map the surface of Mars; (2) learn about past, present, and future efforts and technologies used by NASA to explore the "Red Planet"; (3) understand gravity as a force that keeps the planets in our solar system in their elliptical orbits; and (4) learn how NASA researchers collect, analyze, and interpret the data collected from exploration to develop theories about the existence of water and life on the fourth planet from the Sun.

### CLASSROOM ACTIVITY

Students will conduct a classroom activity to apply what they have learned about geometry and the elliptical paths (orbits) of planets. This activity requires students to use observation, measurement, data collection, and data analysis and interpretation to understand "eccentricity" and how it affects the shape of an ellipse. Students will also calculate the distance between Earth and Mars, the lengths of their respective orbits, and the time it takes the two planets to orbit the Sun. In the video portion of the program, the classroom activity is performed by students from Bridge Street Middle School in Wheeling, WV. For more information on *Geometry of Exploration: Water Below the Surface of Mars?*, visit the NASA CONNECT web site: <http://edu.larc.nasa.gov/connect/water.html>

### WEB-BASED COMPONENT

To further enhance the learning experience, educators and students are invited to access the web-based component of *Geometry of Exploration: Water Below the Surface of Mars?* This online activity, the Mars Millennium WebQuest, supports the AECT/AASL information literacy and national computer/technology standards and supports NASA's Mars Millennium Project <http://quest.arc.nasa.gov/lrc/mars> Developed by Bernie Dodge and Tom March, a WebQuest uses Internet sites to help students learn problem-solving and decision-making skills. Want to know more about WebQuests? Visit <http://www.iste.org/L&L/archive/vol26/no7/features/yoder/index.html>

The Mars Millennium WebQuest is concerned with the future colonization of Mars. In this online activity, students will work individually and

collaboratively, using the Internet as a research tool to gather information about Mars and computer technology to develop an electronic survey to collect and tabulate peoples' opinions about living on Mars. Finally, students participating in the Mars Millennium WebQuest will present their results in the form of a multimedia presentation that will be posted on the NASA CONNECT web site. In the video portion of the program, the technology web-based component is performed by students at Bayside and Brandon Middle Schools in Virginia Beach, VA. You can access the Mars Millennium WebQuest through Norbert's Lab at

<http://edu.larc.nasa.gov/connect/water/norbert/lab.html>

## CAREER CORNER

Access to information is critical when making career decisions. Career Corner, located at <http://edu.larc.nasa.gov/connect/water/ca3.html> features the individuals who appear in *Geometry of Exploration: Water Below the Surface of Mars?* This web site includes pictures of the program participants; summaries of their duties and responsibilities; and includes details about the person, event, or situation that greatly influenced their career choice.

## TEACHER BACKGROUND

### MARS

Humans have known about Mars, the Red Planet, since before recorded history. Throughout the Middle Ages, astrologers studied Mars's motion. Copernicus's theory of 1543 claimed that the planets orbit in circles around the Sun, but it was still unable to predict Mars's motion accurately. It wasn't until 1609 that Johannes Kepler (1571-1630), a German mathematician and astronomer, stated that the orbit of Mars or of any planet is an ellipse.

Most of our current knowledge of Mars is the result of investigations conducted by a fleet of spacecraft beginning with the Mariner mission in the mid-1960's. Among other discoveries, the Mariner mission disclosed evidence of surface erosion and dried riverbeds, indicating that the planet was once capable of sustaining liquid water. This discovery fueled belief in the possibility that life may be (or may have been possible) on Mars. To investigate, two Viking spacecraft were dispatched to Mars in 1975. Many experiments that were performed to test for life showed some interesting chemistry, but no signs of life were found.

Our studies of Mars have left us with many questions: What happened to the atmosphere? Where did the surface water go? The answers to these and other questions about the Red Planet await the next generation of Mars explorers.

In 1996, Mars Pathfinder and Mars Global Surveyor launched the next wave of Mars exploration to study Martian weather, seasonal change, surface features, and composition. These missions set the stage for the Mars Surveyor series, which will send similarly lightweight orbiters and landers to Mars every two years into the first decade of the next century. Orbiters will be data relay stations for the landers, which will probe the soil and test the rocks in search of clues regarding the origins and evolution of the Red Planet, and they will look for tell-tale signs of life forms, past and present. The more we know about Mars, the more we will understand about what has happened and may happen on Earth.

Mars can be seen, at times, in the night sky. Sky charts are available that can help you spot Mars. For example, on December 9, 1999, Mars should be visible in the southwest sky, above and to the left of the Moon. For more information about Mars and the Mars Missions, visit <http://quest.arc.nasa.gov/mars/>

## GEOMETRY AND PLANETARY ORBITS

As stated in the previous paragraph, Johannes Kepler affirmed that the orbit of Mars or of any planet is an ellipse with the Sun at one focus. (The other focus is an imaginary point—there is nothing there.) When we think of the Earth or Mars orbiting the Sun, we generally think of that orbit as being in the shape of a circle. However, the orbits of the Earth, Mars, and all the other planets in the solar system are actually in the shape of a "squashed" circle (an ellipse). (See Figure 1.)

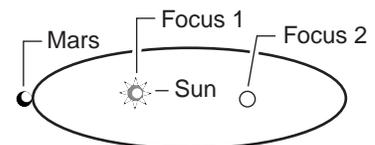


Figure 1

During part of its orbit around the Sun, Mars is closer to the Sun than it is at other times. At its closest, Mars is about 206 million km from the Sun. (See Table A: Solar System Data Chart, page 15). This closest point is called the "perihelion." (See Figure 2.)

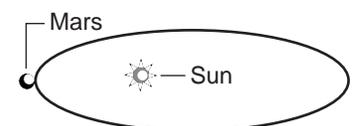


Figure 2

At its farthest, Mars is about 249 million km from the Sun. (See Table A: Solar System Data Chart, page 15). This farthest point is called the "aphelion." (See Figure 3.)

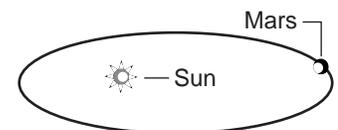


Figure 3

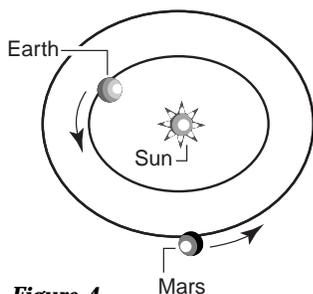


Figure 4

Astronomers often use the average or mean distance from the Sun instead of the minimum or maximum distance.

If we could look down on the Solar System from a location far above the Earth's North Pole, the planets would all be moving around the Sun in a counterclockwise direction. (See Figure 4.)

When launching a rocket to Mars, scientists must shoot from a moving Earth to a moving target. They must aim at where Mars is going to be, not where it is located when the spacecraft is launched. They have found that a straight path is not what is needed. Instead, they launch the spacecraft so that it flies in an elliptical orbit from the Earth to Mars. (See Figure 5.)

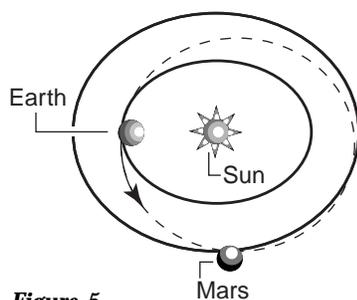


Figure 5

Geometry is very important to NASA scientists and engineers as they plan missions to Mars and other bodies in the Solar System. A knowledge of geometric shapes can help them understand how the planets move and how to send spacecraft from Earth to reach those planets.

## THE ACTIVITY: PLANETARY ORBITS

### NATIONAL MATH STANDARDS

- Number Sense and Operation
- Patterns and Functions
- Geometry and Spatial Sense
- Measurement
- Data Collection and Analysis
- Problem Solving
- Representation
- Reasoning

### NATIONAL SCIENCE STANDARDS

- Science as Inquiry
- Earth and Space Science
- Science and Technology

### NATIONAL TECHNOLOGY STANDARDS

- Basic Operations and Concepts
- Technology-Productivity Tools
- Technology Problem-Solving Tools

## INSTRUCTIONAL OBJECTIVES

Students will

- use measurement and observation to understand the meaning of the eccentricity of an ellipse.
- gather, analyze, and interpret data to determine the distance between the Earth and Mars, the lengths of the orbits of Earth and Mars, and the rate at which the two planets are traveling around the Sun.
- think critically and logically to reach a conclusion about planet orbital rates as compared to their distances from the Sun.
- formulate problems from situations within and outside mathematics.
- apply mathematical thinking and modeling to solve problems that arise in other disciplines.
- verify and interpret results with respect to the original problem situation.
- understand and apply reasoning processes, with special attention to spatial reasoning.
- model situations by using written, concrete, pictorial, graphic, and algebraic methods.
- analyze tables and graphs to identify properties and relationships.
- use content specific tools and software to support learning and research.
- select and use appropriate tools and technology resources to accomplish a variety of tasks and solve problems.

## TEACHER RESOURCES

### Books / Publications

- Beatty, K. , et al.: (1998) *The New Solar System*, 4th Ed., Cambridge, MA: Sky Publishing
- Kastner, Bernice. (1985) *Space Mathematics*. Washington, DC: NASA
- Moore, Patrick. (1990) *The Atlas of the Solar System*. Richmond Hill, NY: Crescent Publishing.
- NCTM (1997). *Mission Mathematics Grades 5-8*. Reston, VA: National Council of Teachers of Mathematics.
- Raeburn, P., (1998) *Mars: Uncovering the Secrets of the Red Planet*. Washington, DC: National Geographic.

### Pamphlets

- NASA Discovering Mars: *Solar System Exploration Division*, NP-1997-02-223-HQ (Visit [http://cass.jsc.nasa.gov/education/ERC/erc\\_books.html](http://cass.jsc.nasa.gov/education/ERC/erc_books.html) to download file)
- NASA Educational Brief: *Exploring Mars*, EB-1999-02-128-HQ (Visit <http://cass.jsc.nasa.gov/expmars/edb/edbbrief.html> to download file)
- NASA Fact Sheet Viking: 7/25/83 DB (Call NASA's JPL - 818-354-5011 to request)

### Wall Chart/Posters

Mars LG-1997-12-481-HQ (Visit <http://spacelink.nasa.gov>, search this number and download file)

Postcards from Mars EW-1997-02-127-HQ

(Visit [http://cass.jsc.nasa.gov/education/ERC/erc\\_posters.html](http://cass.jsc.nasa.gov/education/ERC/erc_posters.html))

### Video

*Space Flight: The Application of Orbital Mechanics* - NASA

*A Man's Reach Should Exceed His Grasp* - NASA

(Search <http://catalog.core.nasa.gov/core.nsf> for these videos)

### Web Sites

#### NASA Spacelink

<http://spacelink.nasa.gov>

#### CORE - Central Operation of Resources for Educators

<http://core.nasa.gov>

#### NASA Online Resources for Educators

<http://www.hq.nasa.gov/office/codef/education>

#### The Math Forum

<http://forum.swarthmore.edu/geometry.k12.geometry.html>

#### The Nine Planets

<http://seds.lpl.arizona.edu/nineplanets/nineplanets.html>

#### Welcome to the Planets

<http://pds.jpl.nasa.gov/planets/>

#### Live from Mars: NASA Ames Research Center

<http://quest.arc.nasa.gov/mars/>

#### "Images of Mars": Space Telescope Science Institute

<http://marvel.stsci.edu/exined-html/Mars.html>

#### NASA Mars Missions

<http://mars.jpl.nasa.gov>

#### Mars Millennium Project

<http://quest.arc.nasa.gov/lc/mars/>

#### Marsville: The Cosmic Village and Mars City Alpha

[http://www.challenger.org/tr/tr\\_prpro\\_set.htm](http://www.challenger.org/tr/tr_prpro_set.htm)

#### Norbert's Lab on the NASA CONNECT web site

<http://edu.larc.nasa.gov/connect/water/norbert/lab.html>



Check out Norbert's Lab!

## BEFORE THE ACTIVITY

Encourage students to research Mars, geometry, and other related topics by using the library, the Internet and the resources listed on pages 5 and 6. Ask students to share their thoughts or write their responses to the following questions:

- Where is Mars?
- How long would it take to get there?
- What do planetary orbits look like?
- Name some geometric shapes and describe how they relate to space and our solar system.
- How fast do planets travel in their orbits? Do they all go the same speed?
- Can a spacecraft go directly from the Earth to another planet?

## VOCABULARY

**Orbit** - the path of one body around another, such as the path the Earth takes around the Sun.

**Ellipse** - a "squashed" circle or closed-plane curve that is formed when the sum of the distances from two fixed points (foci) is constant.

The sum of the solid line segments equals the sum of the dotted line segments. (See Figure 6 below.)

**Perihelion** - the point in its orbit at which a planet is closest to the Sun.

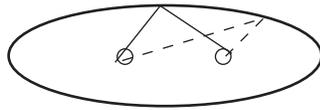


Figure 6. Ellipse

**Aphelion** - the point in its orbit at which a planet is farthest from the Sun.

**Major axis** - the longest distance across an ellipse. (See Figure 7 below.)

**Minor axis** - the shortest distance across an ellipse. (See Figure 8 below.)

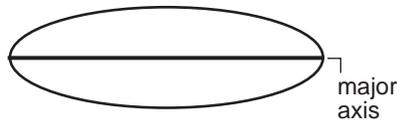


Figure 7. Major axis

**Eccentricity** - the measure of the shape of an ellipse, ranging from zero

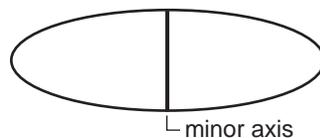


Figure 8. Minor axis

(a circle) to 1 (a straight line).

**Sidereal Period** - length of time needed for a planet to orbit the Sun once.

**Circumference** - distance around a circle;  $2\pi r$

## MATERIALS NEEDED

computer with spreadsheet  
(or calculators)  
classroom copies of  
Table A: Solar System  
Data Chart (page 15)  
Table B: Eccentricity Chart  
(page 16)  
centimeter graph paper  
pushpins - 2 per group  
string - approximately 25 cm  
per group  
cardboard  
metric ruler - 1 per group

## THE ACTIVITY

### Prepare for the Eccentricity Graph activity:

1. Divide the class into 4 research groups.
2. Distribute 1 copy of Table A: Solar System Data Chart (page 15) and 1 copy of Table B: Eccentricity Chart (page 16) to each student.
3. Distribute 1 sheet of centimeter graph paper to each research group.
4. Using Table A: Solar System Data Chart (page 15), explain to the students that charts such as this show the minimum and maximum distances from the Sun to each planet. Ask students to look for relationships between the planets and Sun and discuss their observations.
5. Have students individually calculate the mean distance from each of the planets to the Sun. They may either use a calculator or enter the data from the chart into a spreadsheet. Then ask students the following questions:
  - Which planet in the Solar System has the greatest mean distance from the Sun? (*Pluto*)
  - Which planet has the greatest minimum distance from the Sun? (*Neptune. Pluto's orbit is very elliptical, so at times, it is closer to the Sun than Neptune is.*)
  - Which planet, Mars or Earth, is farther away from the Sun? (*Mars*)
6. On a separate piece of paper, have students make a sketch of the orbits of the Earth and Mars around the Sun. (*See Figure 9 below.*)

### CHALLENGE:

Make a scale model that contains the sizes of the Earth and Mars and their mean distances from the Sun.

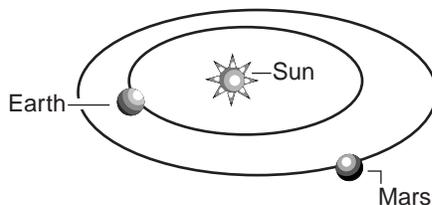


Figure 9

7. Have students look at the Eccentricity (e) column on Table A: Solar System Data Chart (page 15). Eccentricity (e) gives an indication of the "round" or "squashed" appearance of each ellipse. To understand the value of eccentricity, have students perform the following experiment with their team and then compare the results with those of other teams.

→ **Eccentricity activity**

1. On the piece of centimeter graph paper, instruct students to draw two lines—one near the middle vertically and one near the middle horizontally. The lines intersect at the center point. (See Figure 10.)
2. If your students are using standard size graph paper, have them cut a piece of string about 25 cm long. If your students have a larger piece of paper, they can use a longer string. Have students tie a knot near the ends of the string to form a loop. (See Figure 11.)
3. Students should put the graph paper on a piece of cardboard and then place two pushpins along the horizontal line, each 1-centimeter from the center point. These pins represent the foci. At this point, the foci are 2 cm apart. (See Figure 12 below.)

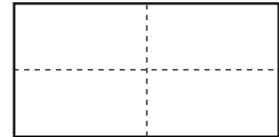


Figure 10



Figure 11

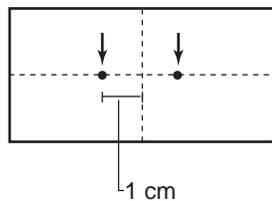


Figure 12

4. Have students loop the string around the pushpins. Next, use a pencil to keep the string taut and draw an ellipse. (See Figure 13 below.)

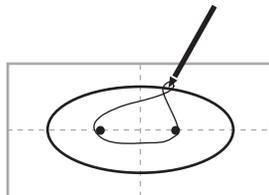


Figure 13

5. Have students measure, in centimeters, the length of the ellipse along its major axis. (See Figure 14 below.)

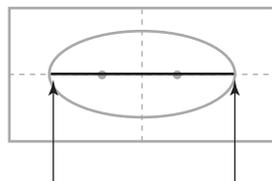


Figure 14

6. Have students record the distance between the two foci (in this case, 2 cm) and the lengths of the major axis on the Table B: Eccentricity Chart (page 16).

**CHALLENGE:**

Consider the eccentricities of the Pluto and Neptune orbits. Compare the minimum distances from the Sun to those two planets. Is Pluto always the most distant planet from the Sun? What does eccentricity have to do with the relationship between the orbits of Neptune and Pluto?

7. Have students divide the distance between the foci by the length of the major axis and record the quotient on the chart. (*This quotient should be a decimal less than 1.*) Next, have students make a sketch of the shape of the ellipse.
8. Students should repeat steps 3-7 using the same string but each time increasing the distance between the foci to 3 cm, 4 cm, 5 cm, and another distance of their choice.

**Note:** In each case, the value of the eccentricity is a decimal less than 1.

9. Allow students to observe how the appearance of the ellipse changes as the eccentricity changes and to compare their team's results with results of other teams. Ask the following questions:
  - Which ellipse looked the most like a circle? (*The first one*)
  - What do you think the shape would look like when the eccentricity is zero? (*A circle*)
  - The orbits of most comets have eccentricities that are closer to 1. What would the shapes of their orbits look like? (*A very squashed, or elongated ellipse*)
  - Look at the eccentricity values on Table A: Solar System Data Chart (*page 15*) Which planet has the most "squashed" orbit? (*Pluto*)
  - Is Earth's orbit more or less round than Mars' orbit? (*More round*)

**Analyzing the Data**

Students should review their observations and data and respond to these questions in their journal or in the form of a classroom discussion:

What is the relationship between the value of the eccentricity and the "round" or "squashed" appearance of the ellipse?

(*The smaller the eccentricity, the rounder the ellipse.*)

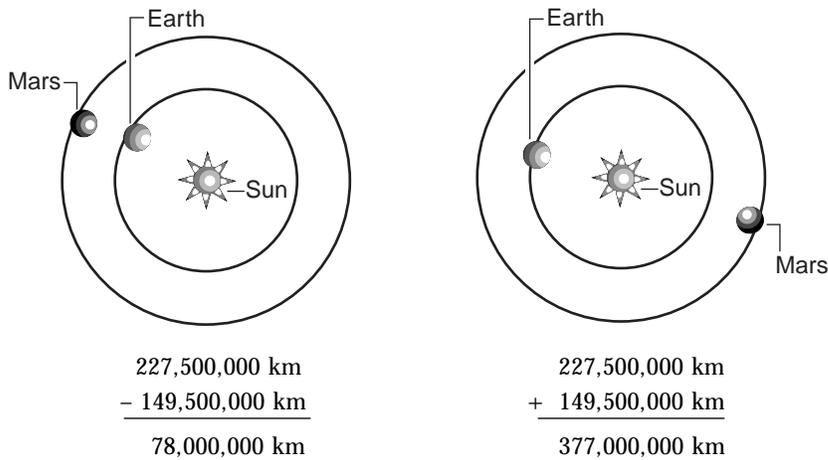
How does the distance between the foci affect the shape of the ellipse?

(*The greater the distance, the more squashed or elongated the ellipse.*)

## EXTENDING THE ACTIVITY

1. Although the orbits of both the Earth and Mars are ellipses, these orbits are so close to being circles that we can make fairly accurate estimates, as though they were exact circles. For example, we can consider the mean distance from the Sun to each planet as the radius of a circle to estimate the distance from the Earth to Mars. This distance can vary greatly, depending upon whether or not both planets are on the same side of the Sun.

Use the mean distances from the Sun to each planet to determine the approximate direct distance between the Earth and Mars in each of the cases below:



2. An Earth year is the length of time it takes the Earth to orbit the Sun, or about 365 days. Astronomers call this the "sidereal period." A Mars "year" takes nearly twice as long - about 687 Earth days. (If you lived on Mars, your age on Mars, measured in "Mars years," would be about half what it is on Earth.) Mars missions only occur about every 2 years. Why is that?

*(Due to the length of their orbits and the time it takes for each planet to make 1 revolution around the sun, it is only every 24 months that Earth and Mars are on the same side of the sun and are fairly in line with each other. (See figure above left.) Every two years is the most efficient time in terms of fuel and energy to launch a rocket from Earth to Mars.)*

2. If Van could drive his van nonstop at a rate of 100 km per hour and go directly from Earth to Mars (when they are both on the same side of the Sun), how long would it take him to get there?

To answer this question, we can use the formula that time equals distance divided by rate:

$$t = d/r.$$

$$Time = \frac{78,000,000 \text{ km}}{100 \text{ km/hr}} = 780,000 \text{ hr} = 32,500 \text{ days} = 89 \text{ years}$$

**CHALLENGE:**

A spacecraft can travel to Mars in about 260 days. If that spacecraft could travel in a straight line to Mars, what would its average rate in kilometers per hour be?

$$r = d/t \quad r = \frac{78,000,000}{260 \text{ days} \times 24 \text{ hrs/day}} = 12,500 \text{ km/hr}$$

3. Of course, driving a van to Mars would not be possible, and until recently, even a rocket could not reach Mars. One problem is that the planets are not standing still but are both moving as they orbit the Sun. How fast are they moving?

Again, we can use the distance-rate-time relationship. To answer this question, we'll rearrange the formula to solve for rate:

$$\text{rate} = \text{distance}/\text{time}$$

The distance traveled is the length of the orbit. To make our computations easier, we'll again consider the orbits to be exact circles. Thus, the distance traveled by each planet in one "year" can be expressed as the circumference of a circle with the distance from the Sun to the planet as the radius. The formula for circumference is

$$C = 2\pi r, \text{ with } \pi \text{ approximately } 3.14$$

What is the circumference of Earth's orbit?

$$C = 2 \times 3.14 \times 149,500,000 = 938,860,000 \text{ km}$$

What is the circumference of Mars' orbit?

$$C = 2 \times 3.14 \times 227,500,000 = 1,428,700,000 \text{ km}$$

Using the sidereal period from the table above for the length of time for one orbit, find the rate each of the planets is moving around the Sun. Are those rates faster or slower than Van's van speed of 100 km/hr? (*Faster*)

Are those rates faster or slower than the average speed needed for a spacecraft to go from the Earth to Mars? (*Faster*)

$$Earth = 938,860,000 \text{ km}/365 \text{ days} = 2,572,218 \text{ km/day} = 107,176 \text{ km/hr}$$

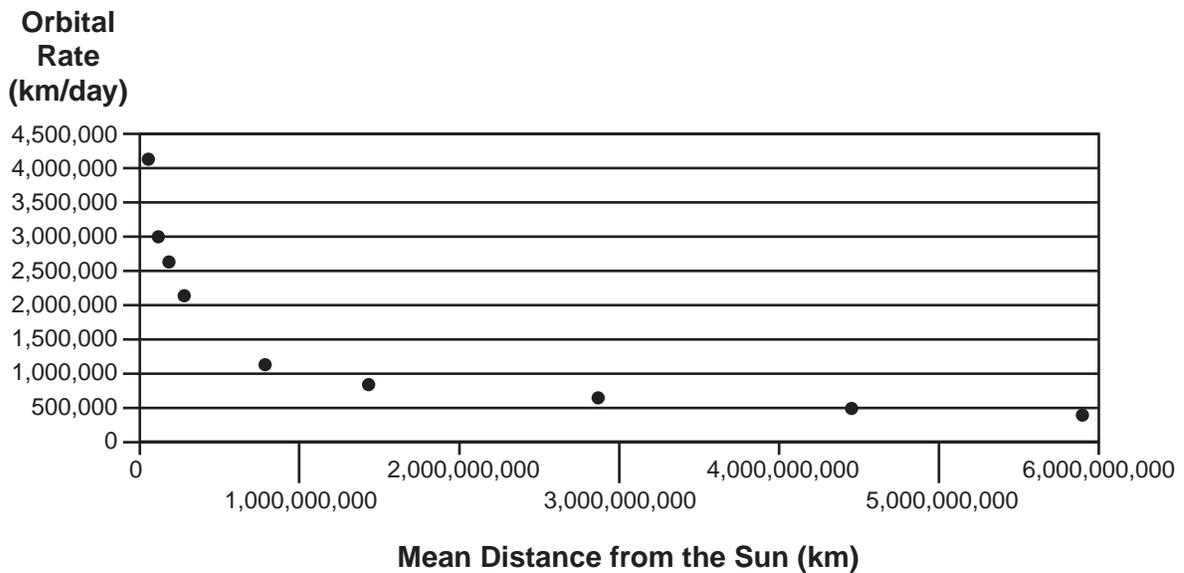
$$Mars = 1,428,700,000 \text{ km}/687 \text{ days} = 2,079,622 \text{ km/day} = 86,651 \text{ km/hr}$$

4. The graph below shows all nine planets with their distances from the Sun and their orbital rates. Which dots on the graph represent the Earth and Mars? (*The third and fourth dots from the left.*) What conclusion can you reach about the orbital rates of all the planets from this graph? (*The greater the distance from the Sun, the slower the orbital rate.*)

**CHALLENGE:**

*Try the Orbit Simulator activity in the NASA/NCTM Mission Mathematics 5-8 publication to experiment with orbit rates.*

DISTANCE FROM THE SUN COMPARED TO ORBITAL RATE



5. For more extension activities, please visit "Norbert's Lab" on the NASA CONNECT web site at <http://edu.larc.nasa.gov/connect/water/norbert/lab.html>



*Encourage students to visit **all** the rooms in Norbert's lab.*

## CUE CARDS, TABLES, CHARTS

### CUE CARDS

**Jennifer and Van, Virginia Air and Space Center**

Who was Pythagoras and what did he contribute to geometry?

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Explain how geometry is used in your everyday life.

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**Dr. Israel Taback, Virginia Air and Space Center**

How do NASA scientists use geometry to navigate spacecraft from Earth to Mars?

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Explain the goals and accomplishments of the Viking mission.

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**Dr. Sarah Gavit, NASA Jet Propulsion Laboratory**

Why are we exploring Mars?

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What tools and techniques does NASA use to explore Mars?

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**Dr. Robert Micheltree, NASA Langley Research Center**

Why do scientists suspect there was once water on Mars? \_\_\_\_\_

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What is the Mars Microprobe and how will it navigate below the surface of Mars? \_\_\_\_\_

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What is the relationship between geometry and the Mars Microprobe? \_\_\_\_\_

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**TABLE A. SOLAR SYSTEM DATA CHART**

<b>Planet</b>	<b>Diameter (km)</b>	<b>Minimum Distance from the Sun (km)</b>	<b>Maximum Distance from the Sun (km)</b>	<b>Mean Distance from the Sun (km)</b>	<b>Eccentricity (e)</b>	<b>Sidereal Period (days)</b>
Mercury	4,878	45,900,000	69,700,000		0.2056	88
Venus	12,104	107,400,000	109,000,000		0.0067	225
Earth	12,756	147,000,000	152,000,000		0.0167	365
Mars	6,794	206,000,000	249,000,000		0.0934	687
Jupiter	142,800	740,900,000	815,700,000		0.0484	4,333
Saturn	120,000	1,347,000,000	1,507,000,000		0.0556	10,759
Uranus	51,118	2,735,000,000	3,004,000,000		0.0472	30,685
Neptune	50,538	4,456,000,000	4,437,000,000		0.0086	60,190
Pluto	2,445	4,425,000,000	7,375,000,000		0.2500	90,465

TABLE B. ECCENTRICITY CHART

Distance Between Foci in Centimeters	Major Axis Length in Centimeters	Eccentricity = Distance Between Foci in cm <hr/> Major Axis Length in cm	Sketch
2 cm			
3 cm			
4 cm			
5 cm			
<hr/> choose a distance			