National Aeronautics and
Space Administration

Langley Research Center
Hampton, VA 23681-0001

NASA CONNECT

Program 6 in the 1999-2000 Series

Proportionality:
Modeling the Future

Story Line: Students will examine how patterns, measurement, ratios, and proportions are used in the research, development, and production of airplanes.

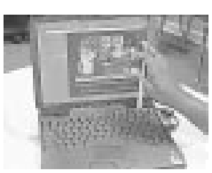
Math Concepts: Computation, Ratios, Measurement, Data Visualization, Patterns

Science Concepts: Force, Motion, Energy, Temperature, Heat, Sound

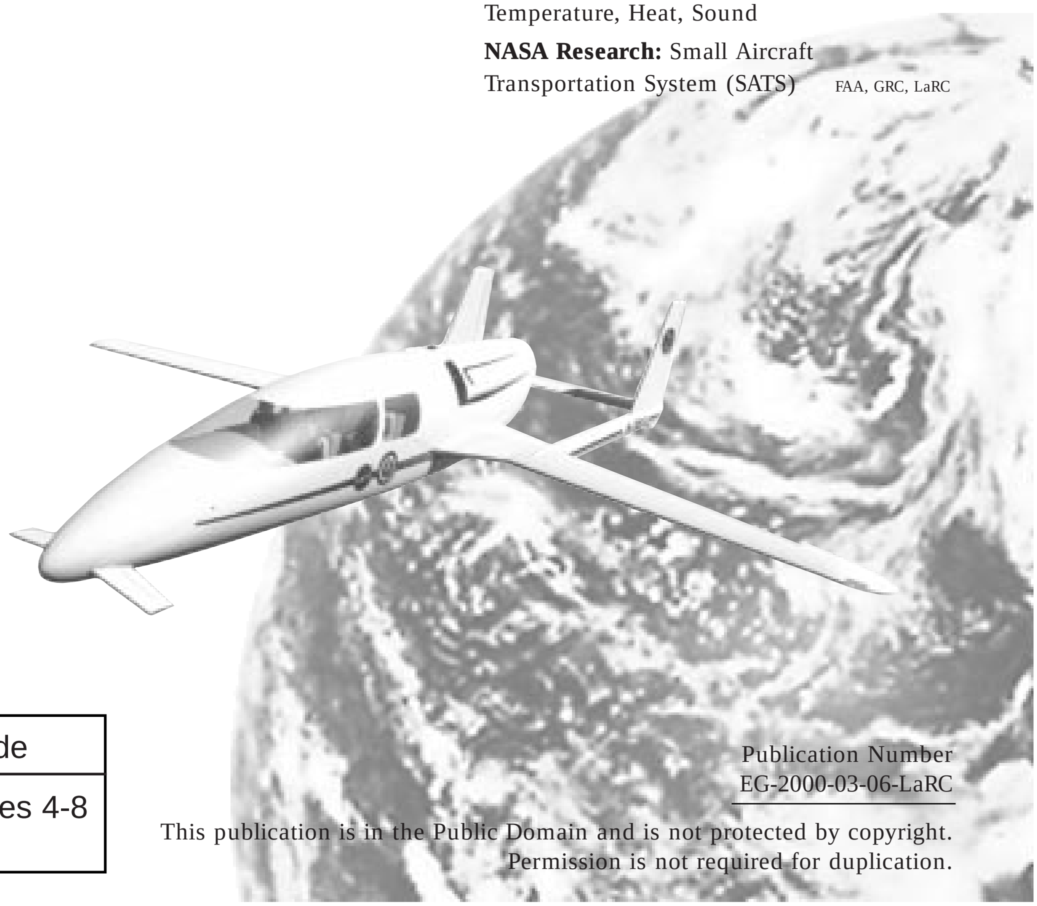
NASA Research: Small Aircraft Transportation System (SATS) FAA, GRC, LaRC

■ *determine The Golden Ratio*

■ *learn how the Wright Brothers used ratios*



■ *use the Internet and visit Norbert's Lab*



Educator's Guide	
Teachers & Students	Grades 4-8

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PROGRAM SUMMARY

OBJECTIVE

In *Proportionality: Modeling the Future,* students will examine how patterns, measurement, ratios, and proportions are used in the research, development, and production of airplanes. Students will meet a pilot from the Federal Aviation Association who will describe the growth of air transportation and its mathematical pattern and a professor who will explain a special pattern of numbers called the Fibonacci sequence. From this sequence, students can calculate the Golden Ratio, a special ratio found in nature, and discover how ratios are used in the design of everyday objects. Students will also see how NASA researchers are using ratios, proportions, and the Golden Ratio to design airplanes and test small aircraft data. To learn more about NASA CONNECT, visit our web site: **edu.larc.nasa.gov/connect**

CLASSROOM ACTIVITY

In this activity, students will use objects found in nature to discover how ratios and proportions are present in everyday objects and their bodies. Students will gain an insight into the mathematics of ratios in nature and how ratios are used in the designs we create. The Fibonacci sequence and the Golden Ratio are used as the basis of discovery.

Before the activity, students should review sequences and determine the succeeding terms of a sequence. Students will examine various natural objects and count petals, sections, or spirals to find numbers in the Fibonacci sequence to verify that the objects are “Golden.”

Students will be asked to calculate the ratios of pairs of numbers within the Fibonacci sequence. They will list the ratios and convert them to decimal form. Students will note the value that the entire sequence of ratios approaches, 1.62 (rounded), which is called the Golden Ratio or Golden Proportion. Next, the students will study the proportions they find by measuring their bodies, by calculating the ratio, and by determining whether they are Golden.

The final activity requires students to examine man-made objects such as buildings, cars, paper objects, and art for the influence of the Golden Ratio.

Suggested time schedule for activity is based on a 45-minute block:

• Preparation for the classroom Activity lesson and collection of materials	Day 1 (45 minutes)
• Classroom Activity	Day 2 (45 minutes)

WEB-BASED COMPONENT

While visiting the corresponding web page for this program, students can access the **Airplane Design Workshop**™, the technology-based component of the program. This online activity is located in *Norbert's Lab* at **edu.larc.nasa.gov/connect/xplane.html.** Desktop Aeronautics, Inc.'s **Airplane Design Workshop**™ is the online activity that provides an opportunity for students to model their own future passenger plane. By choosing different wings, tails, engines, and fuselage layouts, students can use ratios and proportions to design a complete airplane and see if it will fly. With the aid of computer analysis, students will receive quick feedback on the effect of each decision.

CAREER CORNER

Access to information is critical to making career decisions. *Career Corner*, located in *Norbert's Lab* at **edu.larc.nasa.gov/connect/xplane.html**, is a web-based component that highlights the professionals who appear in the program, *Proportionality: Modeling the Future*. This web site includes pictures of the professionals; summarizes their duties and responsibilities; and includes details about the person, event, or situation that greatly influenced their career choice.

TEACHER BACKGROUND

NATIONAL MATH STANDARDS

- Number and Operations
- Patterns, Functions, and Algebra
- Geometry and Spatial Sense
- Measurement
- Data Analysis, Statistics, and Probability
- Communication
- Connections

NATIONAL SCIENCE STANDARDS

- Science and Technology
- Science in Personal and Social Perspectives
- History and Nature of Science

NATIONAL TECHNOLOGY STANDARDS

- Basic operations and concepts
- Social, ethical, and human issues
- Technology communication tools

TEACHER RESOURCES

Books

Brookhart, Clint. (1998) *Go figure!:using math to answer everyday imponderables.* NTC/Contemporary Publishing Group, Inc., Chicago

Garland, Trudi Hammel. (1997) *Fibonacci fun, fascinating activities with intriguing numbers.* Dale Seymour Publications, White Plains

Garland, Trudi Hammel. (1987) *Fascinating Fibonaccis: mystery and magic in numbers.* Dale Seymour Publications, White Plains

Web Sites

How Stuff Works
http://www.howstuffworks.com/gears.htm
Connecting Mathematics and Nature
http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html
Fibonacci Series and The Golden Proportion
http://www.goldenmeangauge.co.uk/fibonacci.htm
WNET SCHOOL Lesson – I AM GOLDEN
http://www.wnet.org/nttidb/lessons/dn/golddn.html
How Divine Is My Proportion
http://www.iit.edu/~smile/ma9lej.html
Fibonacci Sequence in Nature - Ask DR. MATH
http://forum.swarthmore.edu/dr.math/problems/patel12.8.97.html
Fibonacci History
http://www.saumag.edu/art/figure-drawing/vitruvian.html

INSTRUCTIONAL OBJECTIVES

Students will
- identify the successive terms in a given sequence.
- identify Fibonacci numbers and ratios in nature and natural objects.
- use measurement tools to determine body proportions.
- compare their body ratios to the Golden Ratio.
- use measurement tools to determine the linear measurements of man-made objects.
- compare the ratios of man-made objects to the Golden Ratio.

VOCABULARY

ratio – a pair of numbers that is used to make comparisons
proportion – a number sentence or equation stating that two ratios are equal
sequence – a following of one number after another in succession

FIBONACCI RATIO

Early in the 13th century, a mathematician named Leonardo Fibonacci was studying a rabbit problem. Fibonacci wanted to know how many rabbits you would have at the end of the year if you started with a pair of newborn rabbits, one male and one female. Fibonacci knew that newborn rabbits are able to breed after one month and every month thereafter (under ideal circumstances). He found that the sequence 1, 1, 2, 3, 5, 8, 13… demonstrated the total number of rabbit pairs at the end of each month.

Example:
month 1 – the original pair = 1 pair
month 2 – the original pair (now old enough to breed) = 1 pair
month 3 – the original pair + a newborn pair = 2 pairs
month 4 – the original pair and their newborn pair + the first newborn pair = 3 pairs

If you look at the sequence, month 12 corresponds to 144 pairs of rabbits!

Fibonacci and others soon found this sequence occurring in many other things in nature. By counting the spirals of pinecones, pineapples, and sunflower seedheads, you can find neighboring pairs of Fibonacci numbers. The way in which leaves are arranged on a stem also displays a Fibonacci relationship; so do spiral seashells.

The ancient Greeks thought the ratios obtained by successive terms in the sequence were special. This “special” ratio, known as the Golden Ratio, was so pleasing they used it to design their temples and buildings. The Parthenon is an example of a building using rectangles with the Golden Ratio. Objects that can be described by using the numbers in the Fibonacci sequence or the Golden Ratio are said to be in Golden Proportion or Golden.

THE ACTIVITY: THE GOLDEN RATIO

BEFORE THE ACTIVITY

Ask students to recall patterns of numbers in mathematics by providing examples and asking students to provide the next 3 or 4 terms.

Ex. 2, 4, 6, 8, 10…
Ex. 1, 5, 9, 13, 17…
Ex. 3, 12, 48, 192…

Discuss what operations were used to determine the next term.

The following sequence is a very famous sequence called the Fibonacci sequence. What operation is used to determine the terms?

1, 1, 2, 3, 5, 8, 13…

Find the next four terms (21, 34, 55, 89).

The ratio of certain pairs of numbers in the Fibonacci sequence is used to describe things in nature. The ratios look like this:

1/1
2/1
3/2
5/3
8/5
13/8
21/13
34/21
55/34
89/55

If you divide the numerator of each ratio by its denominator, the results look like this:

1/1 = 1
2/1 = 2
3/2 = 1.5
5/3 = 1.666…
8/5 = 1.6
13/8 = 1.625
21/13 = 1.61538…
34/21 = 1.61904…
55/34 = 1.61764…
89/55 = 1.61718…

MATERIALS

Choose a set of four from the following for the first part of the lesson. ***Avoid hybrids as some do not contain Fibonacci numbers.*** *One set per team:*

Pinecones - in good shape, preferably young and still closed, but open ones are okay too
Pineapples
Bananas
Daisies
Grapefruit - white is best
Sunflowers - dried seedheads are easier to count
*Apple*s - cut horizontally between blossom end and stem
Green peppers - cut horizontally to expose the chambers

Choose from the following for part three of the activity:
3 x 5 index cards - one per team
classroom light-switch plate
stamps - cancelled stamps; show variety, not all are golden
paperback novels - one per team
Monopoly™ game board
Front end of a car - may require a "field trip;" some cars are more Golden than others; some are not Golden at all

Other materials needed:
Body Diagram, page 12
Objects Diagram, page 13
Student Data Worksheet, page 11
pencils
calculators
meter stick
string
metric ruler or standard ruler

Do you notice how the ratios begin to get close to the (rounded) number 1.62? The Greeks called the number phi (not to be confused with pi). When something in nature can be described by using the ratios in the Fibonacci sequence, it is said to be Golden. Let's see what we can find that is Golden.

NOTE: Notice that the above ratios are written with the larger number divided by the smaller. If you divide the smaller number in the pair by the larger number, the answer will be .62 (rounded). This value is also recognized as the Golden Ratio; therefore, if a student arrives at a number close to .62 or 1.62, either is acceptable.

THE ACTIVITY

The activity should be done in small groups or pairs with each student counting, calculating, and comparing results with the others in the group. The teacher may choose from the numbered objects given or from the additional natural objects listed, according to availability. Students will record their results on the worksheet provided, page 11.

I. Objective: Be able to identify Fibonacci ratios in nature.
Given the following, the students will examine each object for evidence of the Golden Ratio by identifying Fibonacci ratios.

1. Banana — Students will count the number of sides of the unpeeled banana. Because bananas have either three or five sides, they are said to be Golden.

2. Pineapples — Show the students how to find the spirals and count the number of squares in the spiral. Students will count the number of squares in two adjacent spirals to determine whether they are adjacent numbers in the Fibonacci sequence and therefore ratios; for example, 34 squares in one spiral and 21 in the adjacent spiral. The ratio is 34:21 and therefore the pineapples are Golden. Some pineapples have three different spirals, but all will be Fibonacci numbers.

3. Grapefruit — Students will count the segments of the halved grapefruit. Each grapefruit should yield a Fibonacci number; therefore, a grapefruit is Golden.

ADDITIONAL NATURAL OBJECTS

sunflowers – ratio of petal length to diameter of seed head; number of spirals in two different directions
chambered nautilus – ratio of volume of each cell to the consecutive cell
leaf stem - ratio of distance between one set of leaves to the next set of leaves on the stem
apples - number of seeds inside
green pepper - number of chambers

4. Pinecone — Students will examine the pinecone for the number of spirals that go to the right and compare that number to the number of spirals that go to the left. (See Objects Diagram, page 13.) Use colored markers that will show on the pinecone to keep track of the spirals in each direction. The spirals in one direction should be steeper than the spirals in the other direction. Compare smaller pinecones against larger ones. The ratio should be a Fibonacci ratio. Ex. With 8 spirals to the right and 5 to the left, the ratio is 5:8; therefore, the pinecone is golden.

5. Daisy — Students will count the number of petals that grow in a clockwise direction and the number that grow in a counterclockwise direction. The result is the Fibonacci pair, 21:34; therefore, the daisy is Golden.

II. Objective: Be able to determine body measurements that are approximately the Golden Ratio.

Have students work in pairs or small groups to count and/or measure portions of their bodies as listed. Demonstrate the ratio of finger segments in one finger to the number of fingers on one hand. The ratio should be a Fibonacci ratio, 3:5; therefore, each student's hands are Golden. Students will record results and calculations on their worksheet.

> NOTE: There are several definitions for proportion, depending on the context. When one ratio is equal to another ratio, the equality is called a proportion. When a ratio is equal to the Golden Ratio, it is proportional to the Golden Ratio. (Referred to as the Golden Proportion.)

1. Measure each student's height and record the results on the Student Data Worksheet, page 11. Measure each student from the top of the head to the tip of the middle finger of the outstretched arm; record the results. Compare the ratio of the height to the measure of the length from the top of the head to the end of the outstretched arm. When calculated, does the ratio approximate the Golden Ratio? Ex. 165.5/91.5 = 1.81 (pretty close!)

2. Measure the height of each student and the navel height of each. Write the result as a ratio of *height:navel height*. The result is close to the Golden Ratio.

3. Measure each student's entire arm length and the length of the arm from fingertip to elbow. Write the result as a ratio. The result is close to the Golden Ratio.

III. Objective: Be able to find examples of the Gold Ratio/Proportion in man-made objects.

The teacher may choose objects to be measured based on availability, time, and convenience from the list given at the beginning of the activity or as indicated below.

1. Students will verify the Fibonacci numbers of index cards by measuring the width and the length. The ratio is 5:3 or 3:5; therefore, index cards are Golden.

2. Students will measure cards the size of a credit card. Record the ratio and determine whether the cards are Golden. Encourage students to discuss why something might not be Golden. For example, the card might not fit in a wallet or the card might be cheaper to produce if smaller.

3. Students will measure any of the following to determine if the *length:width* ratio is Golden:
   A. paperback novels
   B. postage stamps
   C. light-switch plates
   D. Monopoly™ game board

4. Students will measure a car grill and a car headlight to determine the Golden Ratio. Is the car designed to be Golden?
- Measure the distance from the center of the car grill to the outside of the headlight.
- Measure the distance from the inside edge of the headlight to the outside edge of the headlight.
- Write the result of the ratio, *largest length:smallest length*.

5. Students will examine the keys on a piano to determine Fibonacci ratios.
- Ratio of *number of white keys:number of black keys*
- The number of black keys in a group

Optional Questions:
Once students have identified the Golden Ratio in certain objects, the following questions can help them use the Golden Ratio in problem solving.
1. A painter wishes to create a mural that would be in Golden Proportion. If the mural is 6.5 feet high, how wide should it be?
2. A quilt is 5.5 feet wide. To have Golden Proportions, what length would it be?

Optional Activity: This activity demonstrates to students how the pleasing ratio of the Golden Proportion influences the things we do.

Ask students to print in upper case letters (as they usually print) the words "GOLDEN RATIOS ARE FUN." Using metric rulers, measure the letters "E", "R", "A", "F" AS FOLLOWS:

- Measure the height of the letter.
- Measure the distance from the middle line to the top of the letter.
- Write the result as a ratio *(long:short)*.
- Measure the other letters in a similar way. Is the students' printing Golden?

Analyzing the Activity

Students should review their observations and respond to these questions in their journals or in the form of a classroom discussion:

In four sentences, describe and analyze the activity.
Was everything you examined "Golden"?
How did you determine whether an object was "Golden"?
Is everything in nature Golden?

Challenge the students to think about the possibility that other special ratios might exist in nature by asking these questions:

Do you think there is another special ratio like the Golden Ratio that exists in nature? Why?
How could someone discover it?
Do all sequences approach one certain number as the Golden Ratio did?

EXTENSION ACTIVITIES

The following activities are meant to be exploratory exercises for students to conduct on their own, as a class activity, or as homework assignments.

1. Have students conduct a search at home for Golden Proportions. Students should list the item and the ratio they find of *length:width*. Prizes could be awarded for the greatest number of items found or for the most unusual list.

2. Have a party in which the only things brought to the party have Fibonacci numbers associated with them. For instance, the fruits in the class activity and/or a Fibonacci ratio of ingredients: *8 tortilla chips:5 Tbsp Salsa* (must show ratio ingredients on a separate paper or card).

3. Students may bring in magazine photos of buildings or monuments to determine their Golden Ratios. (Suggested examples: The United Nations Building, the Parthenon, the Great Wall of China, and the dome on the Capitol building of the United States.)

CUE CARDS

Van Hughes and Jennifer Pulley, NASA Langley Research Center

Use the following information given from the video to set up a proportion.
If your bike wheel makes 1 revolution and travels 239 cm, how many revolutions would your wheel make if you traveled 2352.3 inches? Watch your units.

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How did mathematics and ratios help the Wright Brothers design their glider?

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Arydth Williams, FAA and Jennifer Pulley, NASA Langley Research Center

Describe the growth of transportation since the early 1900's. What is mathematical about its growth?

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Jennifer Pulley and Dr. Bruce Holmes, NASA Langley Research Center

How are NASA engineers using the Fibonacci sequence and the Golden Ratio to research, design, and develop airplanes?

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STUDENT DATA WORKSHEET

NATURAL OBJECTS

	Number of sides or sections
Banana	
Apple	
Grapefruit	

	Ratio of spirals
Pineapple	
Pinecone	
Daisy	

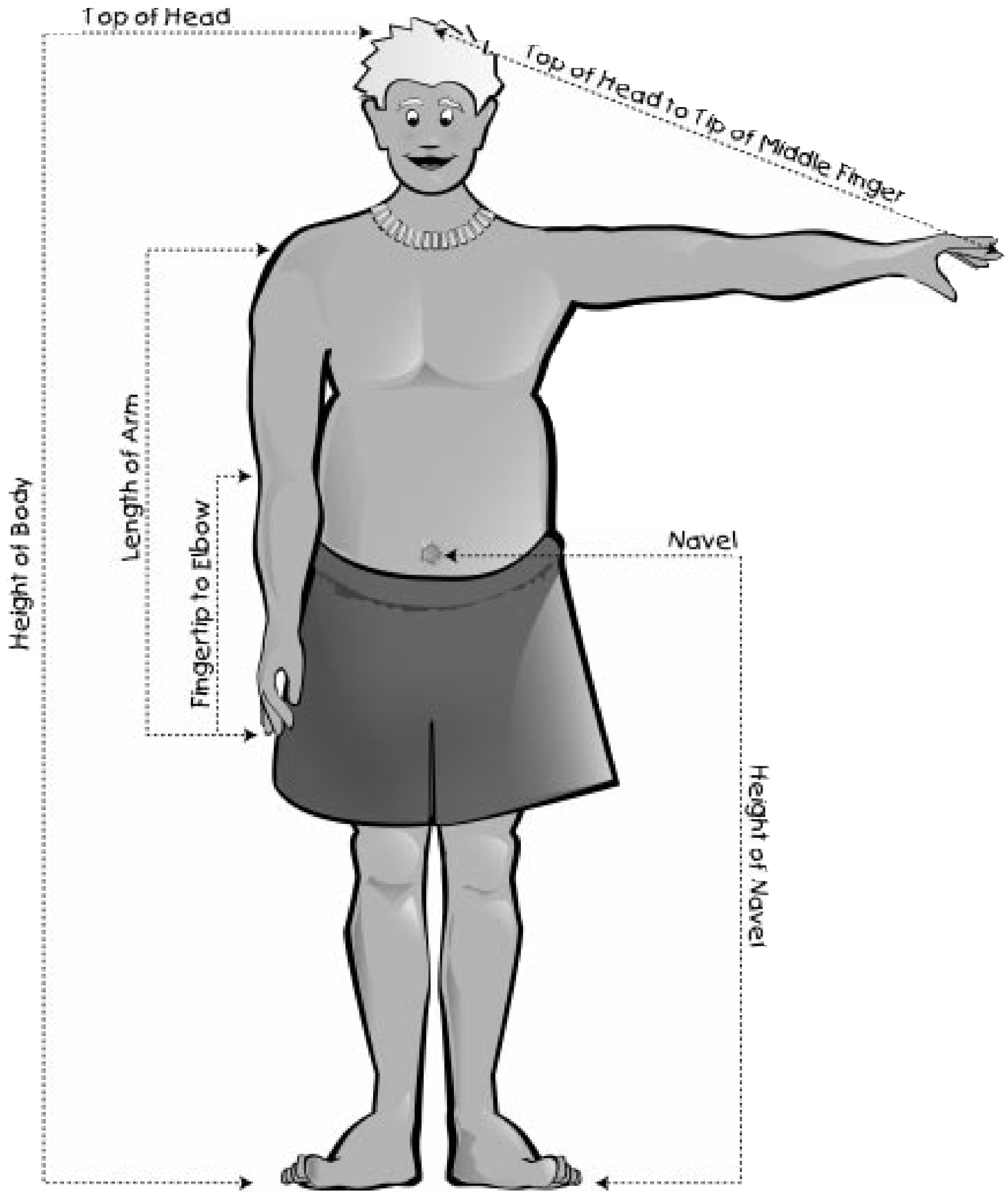
BODY RATIOS

Body measuring	Ratio	Decimal value
Body Height:top of head to tip of finger		
Body Height:height of navel from floor		
Arm length: elbow to fingertip		

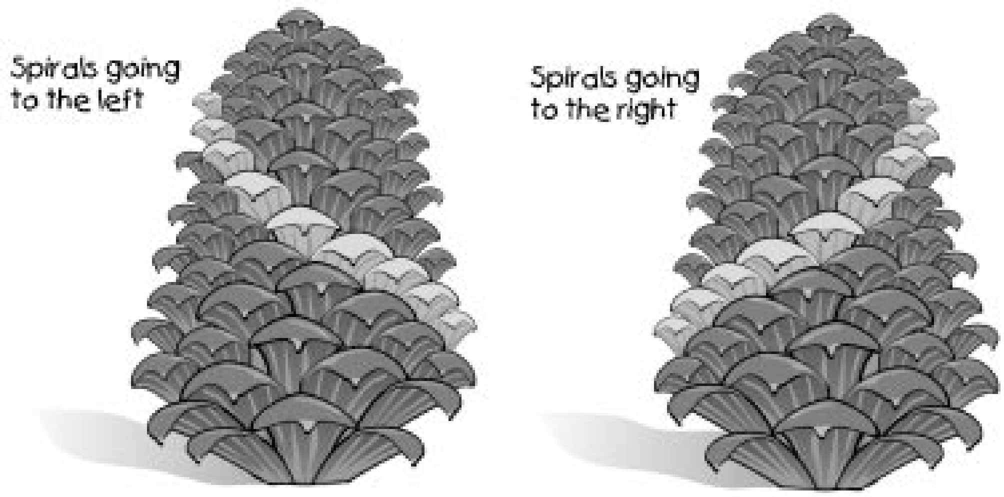
MAN-MADE OBJECTS

Object	Ratio	Decimal value
Index card		
ID or credit card		
Paperback novels		
Light-switch plate		
Car front end		

BODY DIAGRAM



OBJECTS DIAGRAM-PINECONES & CAR GRILL



Pinecones

Car Grill