



National Aeronautics and  
Space Administration  
Langley Research Center  
Hampton, VA 23681-0001



Program 4 in the 1999-2000 Series

# Geometry of Exploration: Eyes Over Mars



■ *use geometry to determine circumference*



■ *use tools and techniques to measure angles and shadows*



■ *learn how surveyors use geometry to measure sports fields*

**Story line:** Students will learn how engineers and scientists are using geometry and linear and angular measurements to survey the Earth and Mars and how geometric shapes affect navigation.

**Math Concepts:** Measurement, Geometric Angles and Shapes, Data Collection and Analysis

**Science Concepts:** Force, Motion, Energy, Space Systems

**NASA Research:** Mars Global Surveyor

Educator's Guide	
Teachers & Students	Grades 4-8

Publication Number  
EG-2000-01-04-LaRC

This publication is in the Public Domain and is not protected by copyright. Permission is not required for duplication.

## PROGRAM SUMMARY

### OBJECTIVES

In the *Geometry of Exploration: Eyes Over Mars*, students will examine how the principles of geometry and linear and angular measurements are used to survey and map the Earth and planets such as Mars. Students will meet a surveyor who will (a) explain how he surveys locations like football or soccer fields, (b) describe the tools and techniques he uses, and (c) show students how math and geometry are used in surveying. Students will also see how NASA researchers use geometric shapes to navigate spacecraft to Mars and how satellites, like the Mars Global Surveyor, and the principles of geometry are used to determine the elevation of land formations on Mars.

### CLASSROOM ACTIVITY

Students will conduct a classroom activity to apply what they've learned about geometry and the relationship between the sides and angles of triangles. This activity requires students to use observation, measurement, collaboration, data collection, and data analysis and interpretation to determine accurate measurements of angles and shadows. This lesson comes from **AskERIC Lesson Plans** (#AELP-GEO0004). ERIC is the Educational Resources Information Center, a federally funded national information system that provides a variety of services and products on a broad range of education-related issues. **AskERIC** is a personalized Internet-based service providing education information to teachers, librarians, counselors, administrators, parents, and others throughout the United States and the world. Visit [askeric.org](http://askeric.org) for more information. To learn more about *Geometry of Exploration: Eyes over Mars*, visit the NASA CONNECT web site: [edu.larc.nasa.gov/connect/wings.html](http://edu.larc.nasa.gov/connect/wings.html)

### WEB-BASED COMPONENT

While visiting the corresponding web page for this program, students can access **Eyes Over Mars**, the technology-based component of the program. This online activity is located in "Norbert's Lab" at [edu.larc.nas.gov/connect/eyes/norbert/lab.html](http://edu.larc.nas.gov/connect/eyes/norbert/lab.html)

The **Eyes Over Mars** online activity provides an opportunity for students to simulate building a Mars Global Surveyor. The students' goal is to accomplish the most complete survey of the Martian surface by using the most appropriate equipment at the lowest budget. Students will make decisions about which type of equipment to use based on descriptions for each apparatus. Each apparatus has benefits and drawbacks which will affect the outcome of the entire mission. When the students have made their choices, the surveyor is built and ready for launch. After the launch, students will receive the results of their mission and compare their results with other participants.

## CAREER CORNER

Access to information is critical to making career decisions. **Career Corner**, located at [edu.larc.nas.gov/connect/eyes/ca2.html](http://edu.larc.nas.gov/connect/eyes/ca2.html), is a web-based component that highlights the professionals who appear in the program, *Geometry of Exploration: Eyes Over Mars*. This web site includes pictures of the professionals; summarizes their duties and responsibilities; and includes details about the person, event, or situation that greatly influenced their career choice.

## TEACHER BACKGROUND

### EUCLIDEAN GEOMETRY

Erastotenes, a Greek mathematician and geographer (who lived from 276 to 194 B.C) made a surprisingly accurate estimate of the Earth's circumference. He read that a deep vertical well near Syene, in southern Egypt, was entirely lit up by the Sun at noon once a year. He reasoned that at this time the Sun

must be directly overhead, with its rays shining directly into the well.

In Alexandria, almost due north of Syene, he knew that the Sun was not directly overhead at noon on the same day because a vertical object cast a shadow. Knowing this, Erastotenes reasoned he could measure the circumference of the Earth by making the assumptions that the Earth was round and that the Sun's rays are essentially parallel.

Using Euclidean geometry, Erastotenes set up a vertical post or gnomon at Alexandria and measured the angle of its shadow at the same time of day that the well at Syene was completely sunlit and no shadow was cast. He knew from geometry and the relationships between angles, parallel lines, and a transversal, that the angle measured in Alexandria was equivalent to the angle at the center of the Earth. By dividing the measurement of a circle, 360 degrees, by the Earth's central angle and then multiplying that quotient by the distance between Alexandria and Syene, Erastotenes determined the circumference of the Earth.

The formula Erastotenes used is

$$\frac{D}{d} = \frac{A}{a} \text{ and Alexandria}$$

d = distance between Syene

A = 360 degrees assumption of round Earth

a = shadow angle of the gnomon

D = to be determined (circumference)

## THE ACTIVITY: SHADOWS & ANGLES

### NATIONAL MATH STANDARDS

- Problem Solving
- Number Sense and Numeration
- Data Analysis
- Geometry and Spatial Sense
- Measurement

### NATIONAL SCIENCE STANDARDS

- Science as Inquiry
- Earth and Space Science
- Science in Personal and Social Perspectives
- Science and Technology

### NATIONAL TECHNOLOGY STANDARDS

- Basic Operations and Concepts
- Technology Research Tools
- Technology Problem-Solving and Decision-Making Tools

### INSTRUCTIONAL OBJECTIVES

Students will

- work effectively in small groups to take accurate measurements at a specific time.
- use knowledge of geometry to determine the size of an angle.
- use significant digits to report their findings.
- research latitude and longitude.
- use research skills to determine accepted values.
- understand the value of cooperation in achieving a common goal.
- research and present results.
- make a scale diagram of activity and compare results.

## TEACHER RESOURCES

### Books / Publications

- Simon, Seymour: (1998) *Mars*, Mulberry Books, N.Y., ISN 0-688-09928-9.  
*Reader's Digest Pathfinders, Space*, Reader's Digest Children Publishing, Inc., N.Y., ISBN 1-57584-291-2.
- Bond, Peter: (1999) *Guide to Space*, DK Publishing, N.Y., Inc., ISBN 0-7894-3946-8.
- Lippincott, Kristen: (1994) *Astronomy*, DK Eyewitness Books, DK Publishing Inc. N.Y., 1994, ISBN 0-7894-4888-2.
- Mitton, Simon and Jacqueline: (1995) *The Young Oxford Book of Astronomy*, Oxford University Press Inc. , N.Y.
- Harrington, Philip and Edward Pascuzzi: (1994) *Astronomy for All Ages*, Globe Pequot Press, CT, ISBN 1-56440-388-2.
- Ulrich, Bertram and Rogers D. Launius: (1998) *NASA and the Exploration of Space*, Stewart, Tabori, and Chang, N.Y., ISBN 1-55670-696-0.

### Web Sites

#### **Starport.com: The Place for Space Explorers**

[www.starport.com](http://www.starport.com)

#### **The Mars Millennium Project**

[mars2030.net/marsinner.htm](http://mars2030.net/marsinner.htm)

#### **Space Kids**

[spacekids.hq.nasa.gov](http://spacekids.hq.nasa.gov)

#### **Welcome to the Planets**

[pds.jpl.nasa.gov/planets/](http://pds.jpl.nasa.gov/planets/)

#### **Mars Education**

[marsnt3.jpl.nasa.gov/education/resources.html](http://marsnt3.jpl.nasa.gov/education/resources.html)

#### **Mars: Mankind's Future Lies in Space**

[hyperion.advanced.org/19455](http://hyperion.advanced.org/19455)

#### **Practical Uses of Math and Science**

[pumas.jpl.nasa.gov/](http://pumas.jpl.nasa.gov/)

#### **Thursday's Classroom**

[www.thursdaysclassroom.com/](http://www.thursdaysclassroom.com/)

#### **ThinkQuest**

[www.thinkquest.org/library/2609.shtml](http://www.thinkquest.org/library/2609.shtml)

#### **NASA Mars Missions**

[mars.jpl.nasa.gov/mgs/](http://mars.jpl.nasa.gov/mgs/)

#### **Mars Team Online**

[quest.arc.nasa.gov/mars/](http://quest.arc.nasa.gov/mars/)

#### **Mars Surveyor Landing Sites**

[marsoweb.nas.nasa.gov/landingsites/](http://marsoweb.nas.nasa.gov/landingsites/)

## BEFORE THE ACTIVITY

Encourage students to research surveying, geometry, Mars, and other related topics by using the library, the Internet, and the resources listed on page 4. Ask students to share their thoughts or write their responses to the following questions:

- What is surveying?
- Name some famous surveyors.
- How is the Earth surveyed?
- How are planets such as Mars surveyed?
- How are shadows created?
- How are shadows used to measure height and distance?

## VOCABULARY

**Acute Angle** - an angle that measures less than 90°.

**Central angle** - an angle whose vertex is placed at the center of a circle.

**Circumference** - distance around the circle.

**Gnomon** - stationary arm whose shadow indicates the time on the sundial.

In this lesson, the gnomon is the stick or pole (i.e., the object that casts a shadow).

**Latitude** - angular distance north or south of the equator, measured in degrees.

**Longitude** - angular distance east or west of the prime meridian, measured in degrees.

**Right triangle** - The hypotenuse is the side opposite the right angle in a right triangle. It is also the longest of the three sides in a right triangle.

The legs are the two sides that form the right angle. (See Figure 1.)

The opposite side is the side directly across from the angle in question.

The adjacent side is the side that shares a common side or vertex with the angle in question.

**Tangent** - refers to the trigonometric function of an “acute angle.” The tangent of an angle is the ratio of the opposite side to the adjacent side.

$$\text{tangent} = \frac{\text{opposite side of triangle}}{\text{adjacent side of triangle}}$$

**Transversal** - in geometry, a transversal is any line that crosses other lines.

**Trigonometry** - comes from a Greek word that means triangle measurement. It is the study of the relationship between the sides and angles of triangles. Trigonometry is used by surveyors, architects, and engineers, as well as by navigators and astronomers.

## MATERIALS NEEDED

straight stick or pole  
(approximately 91 cm)  
meter stick or tape measure  
piece of string  
(approximately 91 cm)  
a rock or weight  
scientific calculator (optional)  
strips of paper  
compass (optional)  
copies of Student Data Chart,  
page 9 (each student)

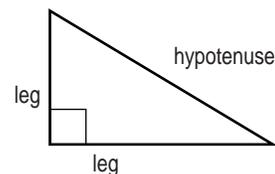
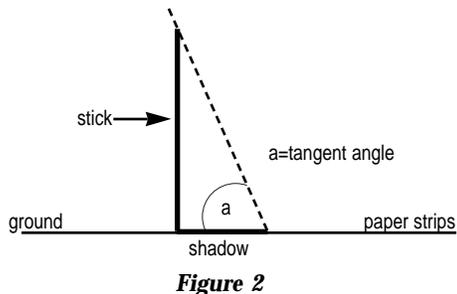


Figure 1

## THE ACTIVITY

In this activity, students will use the same procedure Erastothenes used to measure the angle of the Sun at Alexandria, Egypt.

1. Divide the class into research groups of 3 or 4 members.
2. Set up the measurement station. (See Figure 2, page 6.)



- Place a vertical stick or gnomon perpendicular to the ground. For accurate measurement, it is critical that the gnomon be vertical.
- Have the students dangle a rock tied to a string above the ground in front of the gnomon to ensure that the gnomon is in a true vertical position.
- Consult your local newspaper for the sunrise/sunset times to determine solar noon for your location and time zone.
- Have the students calculate the midpoint.

3. Measure the height of the gnomon.

**Note:** Wind can be a major factor in the activity.

4. Place strips of paper under the station to mark where the shadow ends (See Figure 2).

**Note:** Students may find it interesting that the shadow points towards the north, but use a compass to determine whether the shadow points to true (or magnetic) north.

5. Take measurements of the gnomon's shadow every 2 minutes, beginning at least 10 minutes before local noon, which is the time that the Sun is highest in the sky.

**Note:** This time will probably NOT be 12 noon, as indicated on your time measuring device. Students should note that when the Sun is highest in the sky, the shadow length is the shortest. Since the edge of the shadow is "fuzzy", and the shadow is moving from west to east (in the northern hemisphere), be careful in deciding where to place your mark.

6. Record your data on Student Data Chart 1 (page 9).
7. Locate the latitude and longitude of your school location and record it on Student Data Chart 1 (page 9).
8. Identify your "best" shadow length (the best shadow length at local noon time).
9. Calculate the tangent by dividing the "best" shadow length by the height of the gnomon.
10. Locate this number or the nearest rounded number on the Tangent Table (see page 10). *Option: Find the measure of the tangent by dividing (on a scientific calculator) the length of the shadow by the height of the object.*
11. Record tangent (sun angle) on Student Data Chart 1.
12. Make a scale drawing of your gnomon and shadow. Complete the triangle and measure the tangent (sun angle) with a protractor to verify your calculations. *Option: Repeat the activity at 1:00 pm and 2:00 pm. Compare the data.*

**Note:** By conducting this same activity on the vernal (spring) or autumnal (fall) equinox, the vertical rays of the Sun are directly over the equator, as they were at Syene. By using a globe or an atlas, determine the distance between your location and the equator ( $d$  in equation below) and calculate the Earth's circumference. Use the same formula Erasthenees used:

$$\frac{D}{d} = \frac{A}{a}$$

$d$  = distance between Syene and Alexandria  
 $A$  = 360 degrees assumption of round Earth  
 $a$  = shadow angle of the gnomon  
 $D$  = to be determined (circumference)

### Analyzing the Data

Students should review the results of the activity and respond to these questions in their journals or in the form of a classroom discussion.

- Did the weather conditions affect the results of this activity? If so, how?
- How did you determine your "best" shadow length?
- As the shadow lengthens over time, how will the angle be affected?
- How could you use this math concept to measure the height of other objects? (flag pole, building, and others)
- If each group uses a gnomon with a different length, how will the results be affected?

## EXTENDING THE ACTIVITY

1. Contact a school directly north or south of you (in a different state if possible) and set a specific date and time to take the measurements.
2. Complete Steps 1-9 in the lesson guide.
3. Send your data to your partner school.
4. Enter your data and the data from the partner school on Student Data Chart 2 (page 9).
5. Assign several students to research the circumference of the Earth.
6. Assign other students to determine the distance from your school to the partner school.
7. Subtract the partner school angle from your angle. The absolute value of this difference is the central angle.
8. Record the central angle on the Student Data Chart 2 (page 9).
9. Using the central angle, draw the locations of your school and the partner school on the circumference of a large circle.
10. Calculate the circumference of the Earth by setting up the following ratio:  $\frac{\text{central angle}}{360^\circ} = \frac{\text{distance from school}}{\text{circumference}}$
11. Using the north-south distance to determine the value of the circumference, explain why the ratio and the equation work.

## CUE CARDS

**Terry Tyler, NASA Langley Research Center**

What is surveying? \_\_\_\_\_

How do surveyors use geometry? \_\_\_\_\_

\_\_\_\_\_

**Jennifer Pulley, NASA Langley Research Center**

How did Erasthenes find the circumference of the Earth? \_\_\_\_\_

\_\_\_\_\_

What are the angle relationships between a parallel line and a transversal? \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

**Dan Lyons, NASA Jet Propulsion Laboratory**

What is the Mars Global Surveyor? Where is it? \_\_\_\_\_

\_\_\_\_\_

How does the Mars Global Surveyor use geometry to survey the Martian landscape? \_\_\_\_\_

\_\_\_\_\_

**Dan Lyons, NASA Jet Propulsion Laboratory**

What is aerobraking? \_\_\_\_\_

How does geometry influence aerobraking? \_\_\_\_\_

\_\_\_\_\_

**Corey Hernandez and Brooke Anderson, George Washington University**

How is data, like shadows, collected on Mars? \_\_\_\_\_

\_\_\_\_\_

How is geometry used to determine the height of land formations on Mars? \_\_\_\_\_

\_\_\_\_\_

STUDENT DATA CHART 1

Site (Your School's Name)	Location on Globe (Latitude)	Location on Globe (Longitude)	Shadow Length	Target Angle

STUDENT DATA CHART 2

Site (School Name)	Location on Globe (Latitude)	Location on Globe (Longitude)	Shadow Length	Target Angle	Center of the Earth Angle	North/ South Distance	Circum- ference
#1							
#2							
#3							

TABLE A. TANGENT TABLE

Degree	Tangent	Degree	Tangent	Degree	Tangent
0	0.0000				
1	0.0174	31	0.6008	61	1.8040
2	0.0349	32	0.6248	62	1.8807
3	0.0524	33	0.6494	63	1.9626
4	0.0699	34	0.6745	64	2.0603
5	0.0874	35	0.7002	65	2.1445
6	0.1051	36	0.7265	66	2.2460
7	0.1227	37	0.7535	67	2.3558
8	0.1405	38	0.7812	68	2.4750
9	0.1583	39	0.8097	69	2.6050
10	0.1763	40	0.8390	70	2.7474
11	0.1943	41	0.8692	71	2.9042
12	0.2125	42	0.9004	72	3.0776
13	0.2308	43	0.9325	73	3.2708
14	0.2493	44	0.9656	74	3.4874
15	0.2679	45	1.0000	75	3.7320
16	0.2867	46	1.0355	76	4.0107
17	0.3057	47	1.0723	77	4.3314
18	0.3249	48	1.1106	78	4.7046
19	0.3443	49	1.1503	79	5.1445
20	0.3639	50	1.1917	80	5.6712
21	0.3838	51	1.2348	81	6.3137
22	0.4040	52	1.2799	82	7.1153
23	0.4244	53	1.3270	83	8.1443
24	0.4452	54	1.3763	84	9.5143
25	0.4663	55	1.4281	85	11.4300
26	0.4877	56	1.4825	86	14.3006
27	0.5095	57	1.5398	87	19.0811
28	0.5317	58	1.6003	88	28.6362
29	0.5543	59	1.6642	89	57.2899
30	0.5773	60	1.7320	90	