GEO-PANES

Topic
Geometric figures
Minimum surfaces

Key Question
What kind of geo-pane (soap film pattern) will form on a three-dimensional shape?

Focus
Students will discover and appreciate the unique soap film patterns that form inside polyhedrons.

Guiding Documents
Project 2061 Benchmarks
- *Mathematics is the study of many kinds of patterns, including numbers and shapes and operations on them. Sometimes patterns are studied because they help to explain how the world works or how to solve practical problems, sometimes because they are interesting in themselves.*
- *Use numerical data in describing and comparing objects and events.*

NRC Standard
- *Think critically and logically to make the relationships between evidence and explanations.*

NCTM Standards
- *Describe, model, draw, and classify shapes*
- *Recognize and appreciate geometry in their world*

Math
Geometry and spatial sense
Counting

Science
Physical science
matter

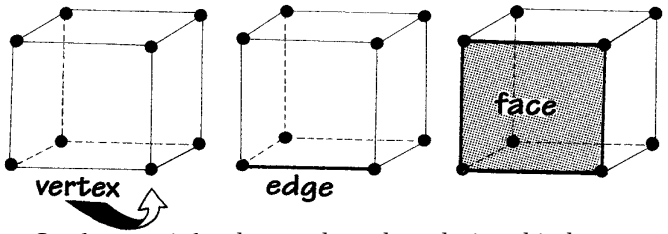
Integrated Processes
Predicting
Observing
Collecting and recording data
Comparing and contrasting
Generalizing

Materials
For the class:
liquid dish soap
1 spool of thread or paper clips
vinegar
1 or more old towels, *optional*
liter and 15 ml measures, *optional*
newspaper and scratch paper, *optional*

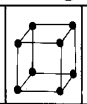
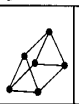
For each group:
clay (see *Management 2*)
toothpicks
1 container such as a half-gallon milk carton

Background Information
Geo-panes are defined as the pattern of *panes*, or soap film, created by dipping shapes into a water and soap solution. Patterns which might be expected to form around the sides of the polyhedrons meet, instead, near the center. The elastic, rubbery skin (surface tension) of the soap film stretches to cover the smallest possible area or minimum surface. Less area is covered when the soap film comes toward the center than if it were to cover the faces around the geometric shape.

The activity begins with the triangle and the square. They have two dimensions, length and width. A third dimension, height, is added when the tetrahedron, cube, triangular prism, and pyramid are built. These three-dimensional shapes are also known as polyhedrons, many-sided figures.



Students might also explore the relationship between vertices, edges, and faces for the polyhedrons they made. The formula, called Euler's (pronounced oilers) Theorem, states that **vertices + faces = edges + 2**. This is true for all convex polyhedrons.These are polyhedrons whose frameworks, if covered, have no indentations.

	2-D		3-D			
Shapes						
# of Vertices (points)	3	4	4	8	6	5
# of Edges (line segments)	3	4	6	12	9	8
# of Faces (sides)	1	1	4	6	5	5

Management
1. This activity will take about 60 to 90 minutes.
2. All figures are made using whole toothpicks.
3. To anchor the toothpicks at the vertices, use oil-based (plasticine) clay rolled into 1 cm balls. Scratch paper helps protect desks while rolling clay. You may prefer to use raisins or dry legumes (soak several hours to soften) instead of clay.

4. Groups of three or four should build the four polyhedrons shown on the activity sheet, each member being responsible for at least one.
5. Spread newspaper over the dipping area (tables or floor), preferably away from desks. Dipping can be done outside if the air is still.
6. For each container, pour water to a depth of no less than 9 cm, add a good squirt of liquid soap, and stir gently so bubbles do not form. If you prefer, have students measure about one liter of water and 15 ml of soap for each container. Another 15 ml of granulated sugar or glycerin (found in drugstores) may be added to strengthen the solution.
7. Caution students to dip carefully so the surface of the soapy water stays relatively free of foam and bubbles. Skim them off if necessary. The bubbles can make it difficult to see the pattern or can actually change it.
8. Students will want to experiment with more complex designs of their own. Suggest they use smaller sections of toothpicks for these.
9. To cut through the soapy film when cleaning up, sprinkle some vinegar on the wet areas and rub dry.

Procedure

1. Give each student a small lump of clay, about 12 toothpicks, about 50 cm of thread, and the activity sheet.
2. Have each student build a square or triangle with the toothpicks and clay. They should record the number of vertices and edges.
3. Borrow a triangle and dip it in the soapy water. Explain that, for this activity, the resulting *pane* will be called a geo-pane. Students should record that it has one face or flat surface. Repeat with the square. Students might notice that both two-dimensional shapes have one face.
4. Ask students to predict what will happen when they dip a three-dimensional shape in the soapy water. Give students a chance to verbalize their ideas before writing their predictions.
5. Instruct students to use their triangle or square to build one of the three-dimensional shapes. Each group should decide who will build which shape so that all of the polyhedrons shown are represented.
6. Have students record the number of vertices, edges, and faces for these shapes BEFORE they go anywhere near the soapy water. Help them determine the number of faces, if needed, as this may be a new term to them.
7. Show students how to slip the thread under one, and only one, toothpick and hold the thread by both ends as shown on the activity sheet. Do not tie the thread to the toothpick.
8. Students should carry their shapes to the dipping area and take turns completely submerging them in the soapy water. They should dip each shape several times to see if the pattern stays the same. Students will likely want to build and try additional shapes.
9. Hold a concluding discussion and have students write about what happened.

Discussion

1. Which of the three-dimensional shapes started with a triangular base? [tetrahedron, maybe the triangular prism (Don't insist students use these terms.)] Which started with a square base? [cube, pyramid]
2. What other shapes did you build and test?
3. Is the geo-pane pattern the same each time? [usually, but a change in conditions – soap bubbles, air currents, etc. – can cause differences]
4. How could you change the geo-pane pattern? [change the thread position, pop one pane, blow on it, etc.]
5. How did you feel when you saw the geo-panes? Which was your favorite geo-pane pattern? (Students may not be able to pick just one.)
6. What other shapes would you like to try?

Extension

Challenge students to study the three-dimensional section of the table to find a relationship between vertices, edges, and faces (see *Background Information*).

Curriculum Correlation

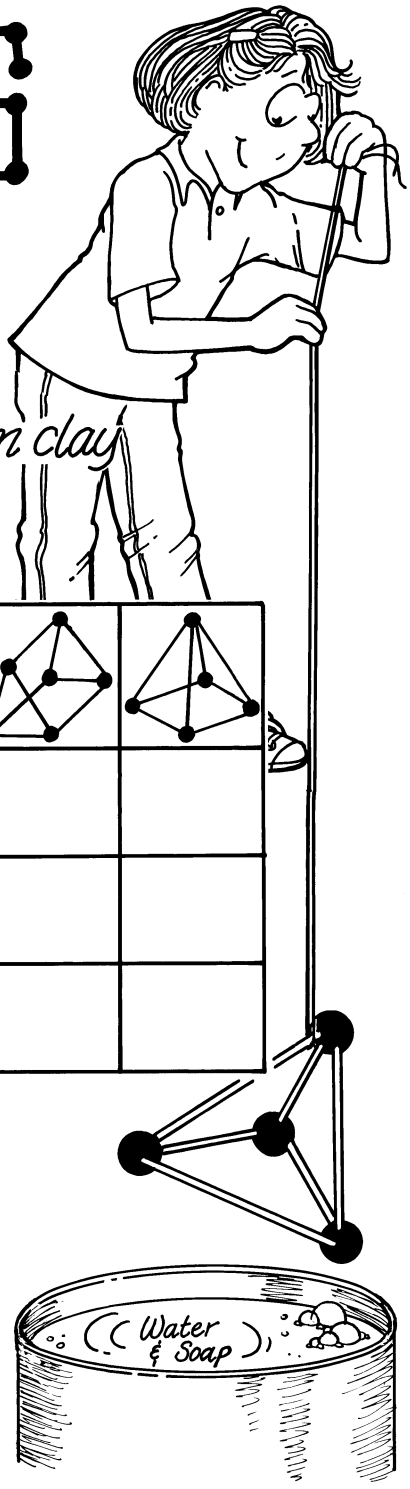
Art

Have students construct polyhedrons such as cubes, tetrahedrons, and pyramids using paper patterns and glue. This is also a good preliminary activity.

Technology

Students might create three-dimensional geometric figures on the computer using *Logo*.

GEO-PANES



What kind of Geo-pane can be made with a 3-D shape ?

1. *Build these shapes with toothpicks and 1 cm clay balls. Fill in the chart below:*

Shapes						
# of Points-Vertices (clay balls)						
# of Line Segments-Edges (toothpicks)						
# of Faces (sides)						

2. *Hang each shape from a thread and dip completely into a water and soap mixture. Lift it out and observe.*

* *What do you think will happen ?*

_____

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_____

* *Describe what happened:* _____

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